2017

Hurlstone Agricultural High School
HSC Assessment Task4 – Trial

Mathematics

Examiners

- **Mr S Faulds**
- **Ms P Biczo**
- **Ms D Crancher**
- **Mr G Huxley**
- **Ms T Tarannum**
- **Ms M Sabah**

General Instructions

- Reading time – 5 minutes
- Working time – 180 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use

In Questions 11 to 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–4)

- Attempt Questions 1 to 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5–14)

- Attempt Questions 11 to 16
- Allow about 2 hours and 45 minutes for this section

Student Name: ______________________________

Class Teacher: ______________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2017 HSC Mathematics Examination.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10. This answer sheet is attached to the back of your examination paper. It may be removed and handed in with your answer booklets for Section 2.

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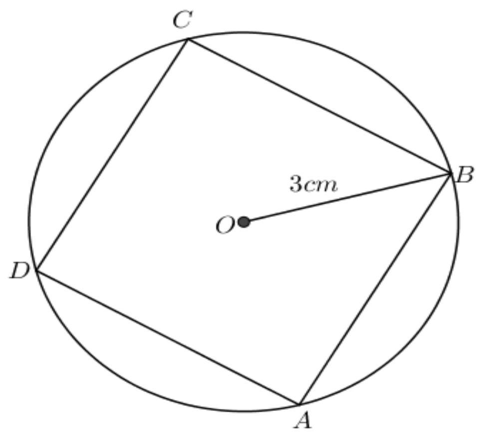
1. How many solutions are there to $\cos 2\theta = \dfrac{\sqrt{3}}{2}$ within the interval $0° \le \theta \le 360°$?

A: 1 **B:** 2 **C:** 3 **D:** 4

2. A square is inscribed in a circle of radius 3 cm as shown.



NOT TO SCALE

What is the area of the square?

A: 81 cm^2 **B:** 36 cm^2 **C:** 9π cm^2 **D:** 18 cm^2

3. Which inequality defines the domain for $y = \dfrac{1}{\sqrt{x^2 - 9}}$?

A: $x < -3$ or $x > 3$ **B:** $x \le -3$ or $x \ge 3$ **C:** $-3 < x < 3$ **D:** $-3 \le x \le 3$

4. The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β.
What is the value of $\alpha\beta + \left(\alpha^2 + \beta^2\right)$?

A: –10 **B:** –8 **C:** 10 **D:** 8

5. What is the value of $\int_1^4 |x-3|\,dx$?

A: 1.5 **B:** 2.5 **C:** –1.5 **D:** –2.5

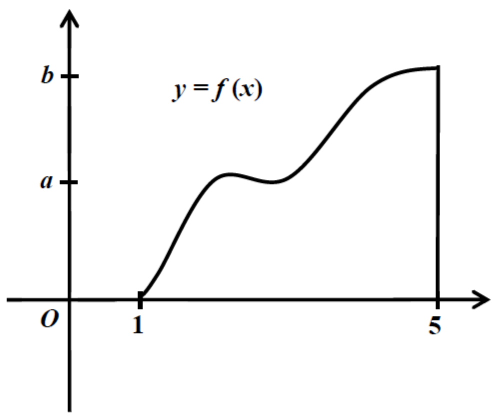
6. Which of the following trigonometric expressions is equivalent to $\tan\left(\frac{\pi}{2}-\theta\right)$?

A: $\tan\theta$ **B:** $-\cot\theta$ **C:** $-\tan\theta$ **D:** $\cot\theta$

7. The equation of the line passing through the point $(2,-4)$ with a gradient of $\frac{1}{2}$ is given by which equation?

A: $y=\frac{1}{2}x-4$ **B:** $y=\frac{1}{2}x-5$ **C:** $y=\frac{1}{2}x+3$ **D:** $y=\frac{1}{2}x+4$

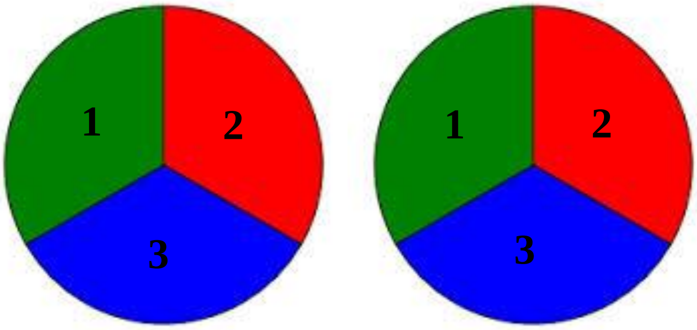
8.



Using Simpson's rule with 3 function values, which expression **best** represents the area bounded by the curve $y=f(x)$, the x-axis and the lines $x=1$ and $x=5$?

A: $\frac{2}{3}(1+4a+b)$ **B:** $\frac{1}{2}(1+4a+b)$ **C:** $\frac{2}{3}(4a+b)$ **D:** $\frac{1}{2}(4a+b)$

9.



Two identical spinners, containing the values 1, 2, and 3 are spun and the results on each are added together. What is the probability that the resulting sum is an even number?

A: $\frac{1}{3}$ **B:** $\frac{5}{9}$ **C:** $\frac{2}{3}$ **D:** $\frac{7}{9}$

10. An infinite geometric series has a first term of 2 and a limiting sum of $1 \cdot 5$.
What is the common ratio?

A: $-0 \cdot \dot{3}$ **B:** $-0 \cdot \dot{6}$ **C:** $-1 \cdot 5$ **D:** $-3 \cdot 75$

Section II starts on the next page.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Start a new answer booklet.** **Marks**

(a) The surface area of a cube is 36 cm^2.
What is the edge length of the cube correct to 3 significant figures? **2**

(b) Simplify the following expression, giving your answer in simplest exact form with a rational denominator: **2**

$$\frac{\sqrt{3}}{2\sqrt{7}-2}$$

(c) Solve the equation: $|x-2| = 3x+1$ **3**

(d) Fully factorise: $2x^4 - 32$ **2**

(e) Find the value/s of x where the graphs of $x^2 + y^2 = 16$ and $y = -\frac{\sqrt{7}}{3}x$ intersect. **2**

(f) Solve the exponential equation, giving your answer correct to 2 decimal places: **2**

$$3^x = 4$$

(g) Use a suitable substitution to solve the following equation: **2**

$$3x^4 - 11x^2 - 4 = 0$$

Question 12 (15 marks) **Start a new answer booklet.** **Marks**

(a) A function is defined as:

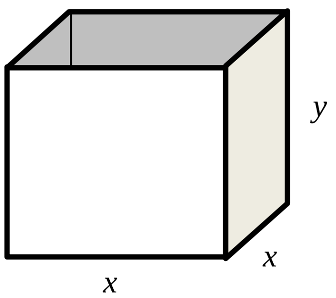
$$\begin{cases} f(x) = 2x - 1 & \text{for } 0 \le x \le 3 \\ f(x) = \frac{1}{3}x + 4 & \text{for } 3 < x \le 5 \end{cases}$$

(i) What is the range of this function? **1**

(ii) Find the value of $f(4) - f(2)$. **1**

(b) Differentiate $(2x+1)^8$ with respect to *x*. **2**

(c) A rectangular box with a square base and no top is drawn below.



The volume of the box is 500 cm^3.

(i) Show that the surface area (A) of the box is given by $A = x^2 + \dfrac{2000}{x}$. **2**

(ii) Find the least area of sheet metal required to make the box. **3**

(d) A parabola has equation $8y = x^2 - 16$.

(i) Find the coordinates of its vertex. **2**

(ii) Find the coordinates of its focus and the equation of its directrix. **2**

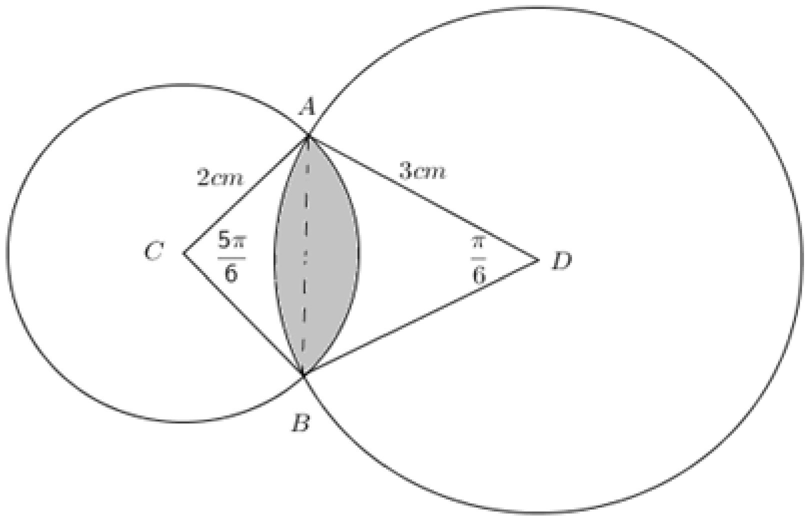
(iii) Sketch the parabola, showing all relevant features. **2**

Question 13 (15 marks) **Start a new answer booklet.** **Marks**

(a) Solve for θ:

$$2\cos^2\theta + 3\sin\theta\cos\theta + \sin^2\theta = 0\,,\ 0^\circ \le \theta \le 360^\circ$$ **3**

(b) In the diagram, two circles with centres C and D intersect at A and B where $AD = 3\text{cm},\ AC = 2\text{cm},\ \angle ACB = \frac{5\pi}{6}$ and $\angle ADB = \frac{\pi}{6}$.



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The shaded region represents the common region of the two circles.

(i) Calculate the perimeter of the shaded region. **2**

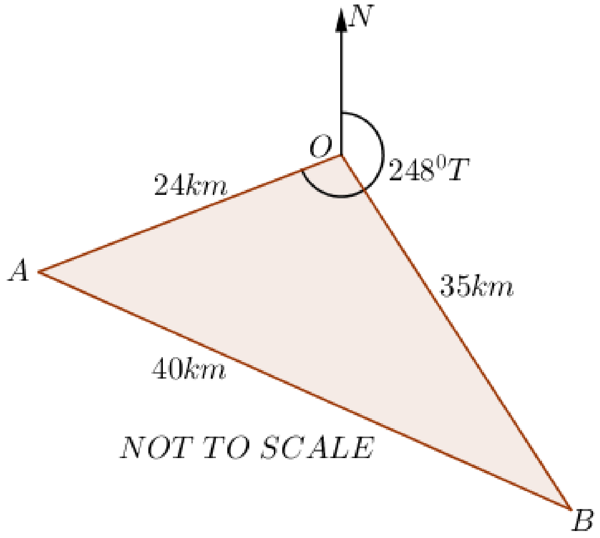
(ii) Calculate the area of the shaded region. **3**

(c) Prove the identity: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$ **2**

Question 13 continues on the next page…

Question 13 (continued) **Marks**

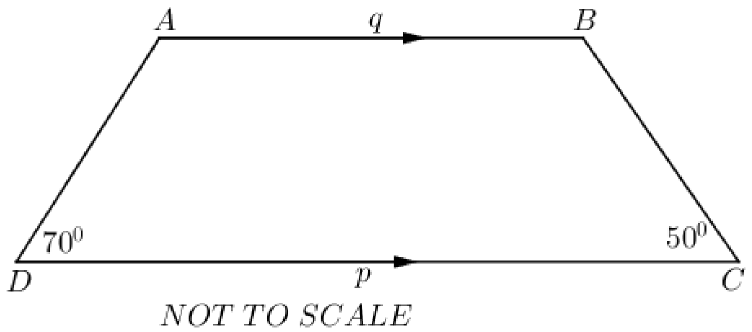
(d) A section of a rainforest is being designated for a species count. The shape is shown below. The bearing of landmark A from landmark O is $248°$T and is 24 km in distance. The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



(i) Show that $\angle AOB = 83^o$, to the nearest degree. **2**

(ii) Calculate the area of the rainforest, correct to the nearest square kilometre. **1**

(e)



In the figure above $AB \parallel DC$, $AB = q$ and $DC = p$.

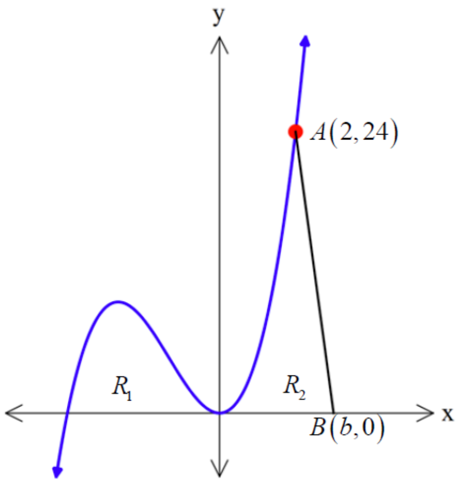
Show that the length of BC is $\dfrac{(p-q)\sin 70°}{\sin 60°}$ **2**

Question 14 (15 marks) **Start a new answer booklet.** **Marks**

(a) Differentiate with respect to x: **2**

$$y = 2xe^{3x}$$

(b) The sketch below shows part of the curve with equation $y = x^2(x+4)$.
The finite region R_1 is bounded by the curve and the negative x-axis.
The finite region R_2 is bounded by the curve , the positive x-axis and AB, where $A = (2, 24)$ and $B = (b, 0)$ where $b > 2$.



NOT TO SCALE

(i) Show that the area of R_1 is $\dfrac{64}{3}$ square units. **2**

(ii) If the areas of R_1 and R_2 are equal, find the exact value of b. **3**

(c) Show that $\displaystyle\int_0^5 \frac{3}{2x+5}\,dx = \ln\left(3\sqrt{3}\right)$ **2**

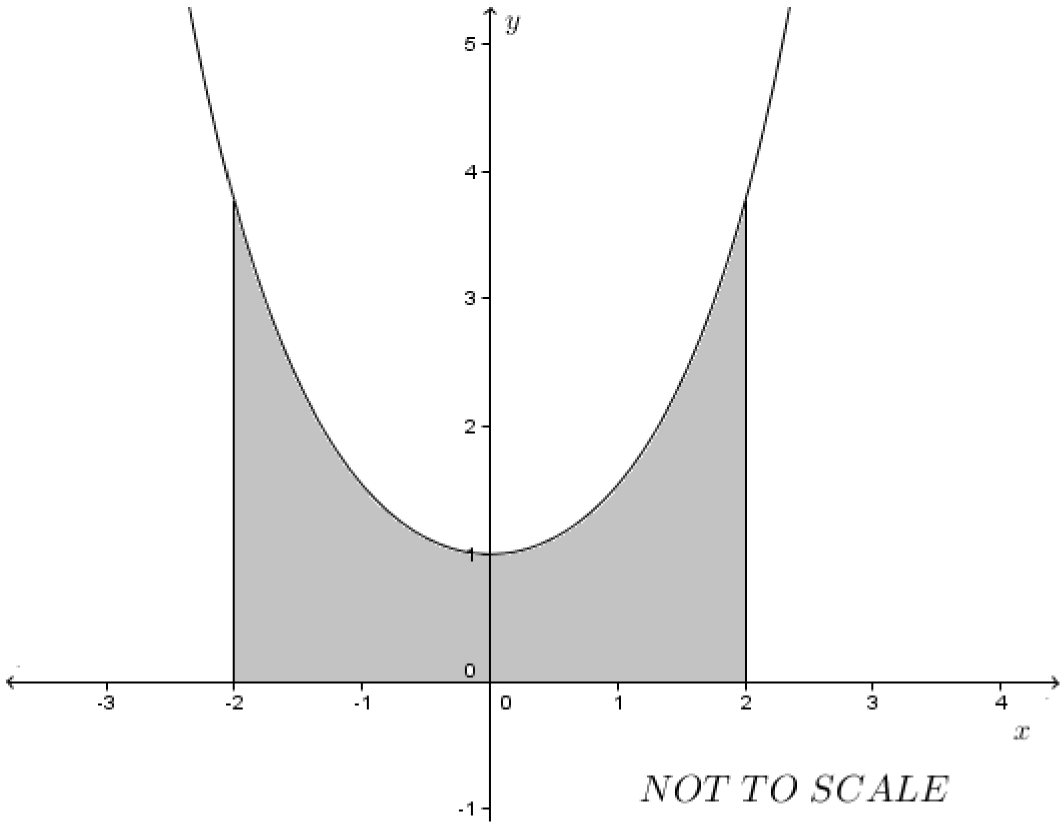
(d) Differentiate $y = (\ln x)^2$ and hence evaluate $\displaystyle\int_1^2 \frac{\ln x}{x}\,dx$ **3**

Question 14 continues on the next page…

Marks

(e) The diagram below shows the graph of the function $f(x)=\frac{1}{2}\left(e^{x}+e^{-x}\right)$. **3**

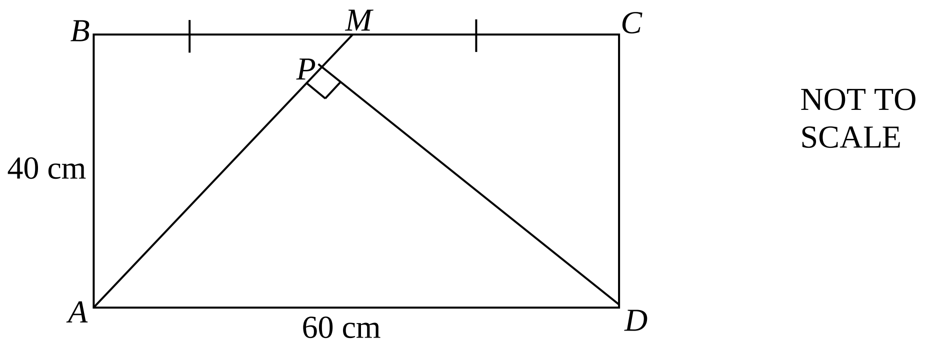
The area bounded by the curve the x-axis and the lines $x = -2$ and $x = 2$ is shaded.



Calculate the volume of the solid of revolution when this area is rotated about the x-axis. Leave your answer in exact form.

Question 15 (15 marks) **Start a new answer booklet.** **Marks**

(a)

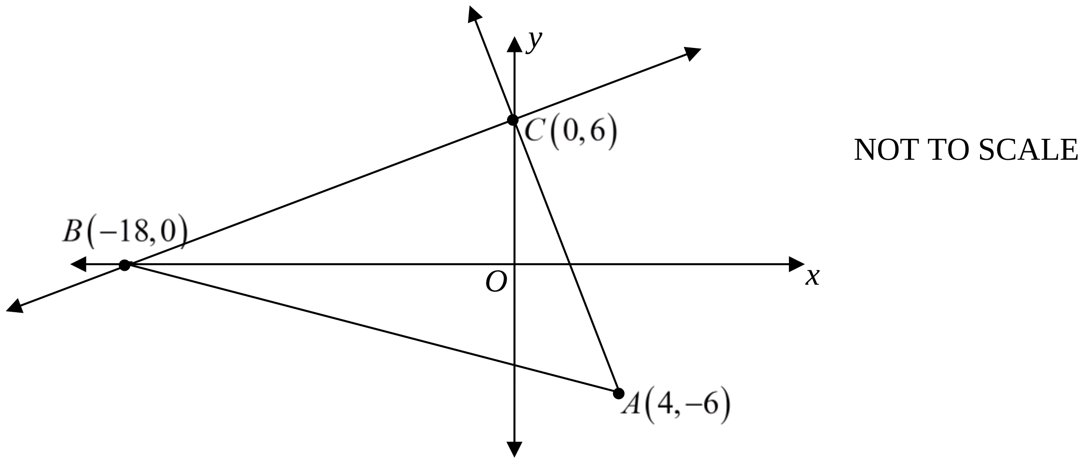


ABCD is a rectangle in which *AB* = 40 cm and *AD* = 60 cm. *M* is the midpoint of *BC* and *DP* is perpendicular to *AM*.

(i) Prove that triangles *ABM* and *APD* are similar. **2**

(ii) Calculate the length of *PD*. **2**

(b) In the diagram, the points *A*, *B* and *C* are $(4,-6)$, $(-18,0)$ and $(0,6)$ respectively.



(i) It is given that the equation of the line *AC* is $3x+y-6=0$. Show that the line *AC* is perpendicular to the line *BC*. **3**

(ii) *AB* is the diameter of a circle which passes through the points *A*, *B* and *C*. Find the equation of the circle. **3**

Question 15 continues on the next page…

Marks

(c) The straight line $y = kx - 4$ is a tangent to the hyperbola $y = \frac{1}{x}$. **2**

Find the value/s of k.

(d) A point $P(x, y)$ moves so that the perpendicular distance of the point to the line $3x - 4y + 1 = 0$ is 2 units.

(i) Find the equation of the locus of P. **2**

(ii) Give a geometrical description of the locus. **1**

Question 16 (15 marks) **Start a new answer booklet.** **Marks**

(a) For the arithmetic sequence 5, 11, 17, 23, …

(i) Write the rule to describe the nth term. **1**

(ii) Find the sum of the first 100 terms. **1**

(b) Two-digit numbers are formed from the digits 2, 3, 4, 5, 6. Repetition of digits is allowed. A two-digit number is then selected at random. What is the probability the number is a multiple of 3? **2**

(c) The fourth and seventh terms of a geometric series are $\frac{15}{2}$ and 60 respectively. What is the first term? **2**

(d) Kylo invests $\$P$ at 7% per annum compounded annually.
He plans to withdraw \$5000 at the end of each year for eight years to cover university fees.

(i) Write down an expression for the amount $\$A_1$ remaining in the account following the withdrawal of the first \$5000. **1**

(ii) Find an expression for the amount $\$A_3$ remaining in the account after the third withdrawal. **2**

(iii) How much does Kylo need to invest if the account balance is to be \$0 at the end of the eight years? **2**

Question 16 continues on the next page…

Marks

(e) A game is played in which two coloured dice are thrown once.
The six faces of the red die are numbered 1, 3, 5, 7, 9 and 11.
The six faces of the white die are numbered 2, 4, 6, 8, 10 and 12.
The player wins if the number on the red die is larger than the number on the white die.

(i) Show that the probability of the player winning a game is $\frac{5}{12}$. **1**

(ii) What is the probability that the player wins exactly once in two successive games? **2**

(iii) What is the probability that the player wins at least once in two successive games? **1**

END OF EXAMINATION

Student's Name: ________________________________

Class Teacher: ________________________________

2017	Hurlstone Agricultural High School **HSC Assessment Task4**

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ✗ B ✗ (correct) C ○ D ○

Start Here

1	A ○	B ○	C ○	D ○	**6**	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○	**7**	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○	**8**	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○	**9**	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○	**10**	A ○	B ○	C ○	D ○

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P3 **performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions** and trigonometric identities

P4 **chooses and applies appropriate arithmetic, algebraic,** graphical, trigonometric and geometric **techniques**

H3 **manipulates algebraic expressions involving** logarithmic and **exponential functions**

Outcome	Solutions	Marking Guidelines
P4	**(a)** Surface area $= 36\text{cm}^2$ Area of single face $= \frac{36}{6}\text{cm}^2$ $= 6\text{cm}^2$ Edge length $= \sqrt{6}\text{cm}^2$ $= 2.45\text{cm}^2$ (3 sig. figs.)	**2 marks** Correct answer with correct rounding. **1 mark** One of the answer or rounding is correct.
P3	**(b)** $\frac{\sqrt{3}}{2\sqrt{7}-2} = \frac{\sqrt{3}}{2\sqrt{7}-2} \times \frac{2\sqrt{7}+2}{2\sqrt{7}+2}$ $= \frac{2\sqrt{21}+2\sqrt{3}}{4\times 7-4}$ $= \frac{2\sqrt{21}+2\sqrt{3}}{24}$ $= \frac{\sqrt{21}+\sqrt{3}}{12}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution including correctly rationalising the denominator.
P4	**(c)** $\lvert x-2\rvert = 3x+1$ $x-2 = 3x+1$ *OR* $-x+2 = 3x+1$ $-3 = 2x$; $1 = 4x$ $x = -\frac{3}{2}$; $x = \frac{1}{4}$ Checking solutions: $\left\lvert -\frac{3}{2}-2\right\rvert \neq 3\times -\frac{3}{2}+1$; $\left\lvert \frac{1}{4}-2\right\rvert = 3\times\frac{1}{4}+1$ $\left\lvert -\frac{7}{2}\right\rvert \neq -\frac{7}{2}$; $\left\lvert -\frac{7}{4}\right\rvert = \frac{7}{4}$ $\therefore x = \frac{1}{4}$ is the only valid solution.	**3 marks** Correct solution, explicitly showing the checking of possible solutions. **2 marks** Correct solution but answers not checked OR substantial progress towards correct solution with possible solutions checked. **1 mark** Some progress towards a correct solution.
P4	**(d)** $2x^4 - 32 = 2\left(x^4 - 16\right)$ $= 2\left(x^2-4\right)\left(x^2+4\right)$ $= 2\left(x-2\right)\left(x+2\right)\left(x^2+4\right)$	**2 marks** Correct factorisation. **1 mark** Substantial progress towards a correct factorisation.

P4	**(e)** Graphs intersect when: $$x^2+\left(-\frac{\sqrt{7}}{3}x\right)^2=16$$ $$x^2+\frac{7x^2}{9}=16$$ $$9x^2+7x^2=144$$ $$16x^2=144$$ $$x^2=9$$ $$x=\pm 3$$	**2 marks** Correct solution giving both possible answers. **1 mark** Substantial progress towards a correct solution.
H3	**(f)** $$3^x=4$$ $$\log 3^x=\log 4$$ $$x\log 3=\log 4$$ $$x=\frac{\log 4}{\log 3}$$ $$=1.26\ (2\text{ dec. pl.})$$	**2 marks** Correct solution. Rounding not important. **1 mark** Substantial progress towards a correct solution.
P4	**(g)** Let $X=x^2$ $$\therefore 3X^2-11X-4=0$$ $$3X^2-12X+X-4=0$$ $$3X(X-4)+(X-4)=0$$ $$(X-4)(3X+1)=0$$ $$X=4,-\frac{1}{3}$$ But, $X=x^2$ $$\therefore x^2=4,-\frac{1}{3}$$ $$=\pm 2\text{ only}\left(\frac{1}{\sqrt{-3}}\text{ has no solutions}\right)$$	**2 marks** Correct solution. **1 mark** Substantial progress towards a correct solution.

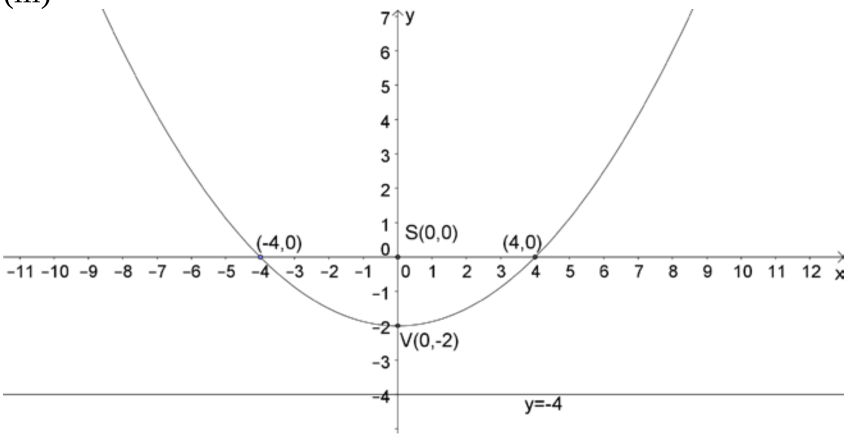
Year 12 2017	Mathematics	Task 4 Trial
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P4 - chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques
H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
P5 - understands the concept of a function and the relationship between a function and its graph
H7 - uses the features of a graph to deduce information about the derivative
P7 - determines the derivative of a function through routine application of the rules of differentiation
P8 - understands and uses the language and notation of calculus
H9 - communicates using mathematical language, notation, diagrams and graphs

Outcome	Solutions	Marking Guidelines
P5, H9	(a) (i) $-1 \le y \le \frac{17}{3}$	1 mark for correct answer
P5, H9	(ii) $f(4) - f(2)$ $= \left(\frac{1}{3} \times 4 + 4\right) - (2 \times 2 - 1)$ $= \frac{7}{3}$	1 mark for correct answer
P7, H5, P8, H9	(b) $\frac{d\left[(2x+1)^8\right]}{dx}$ $= 8(2x+1)^7 \times 2$ $= 16(2x+1)^7$	2 Marks for complete correct solution 1 Mark for substantial work that could lead to a correct solution
P8, H5, H7, H9	(c) (i) $500 = x^2 y$ $\therefore y = \frac{500}{x^2}$ $A = 4xy + x^2$ $A = 4x\left(\frac{500}{x^2}\right) + x^2$ $A = \frac{2000}{x} + x^2$	2 Marks for complete correct solution 1 Mark for substantial work that could lead to a correct solution

P8, H5, H7, H9	(ii) Least value when $A' = 0$ and $A'' > 0$ $A' = -2000x^{-2} + 2x$ $= 2x - \frac{2000}{x^2}$ Now, $2x - \frac{2000}{x^2} = 0$ $2x^3 = 2000$ $x^3 = 1000$ $x = 10$ Now, $A'' = 4000x^{-3} + 2$ $= \frac{4000}{x^3} + 2$ Now, When $x = 10$ $A'' = \frac{4000}{10^3} + 2$ $= 4 + 2$ $= 6 > 0$ Therefore, least area occurs when $x = 10$ $A = \frac{2000}{10} + 10^2$ $= 300\text{cm}^2$	3 Marks for complete correct solution 2 Marks for substantial correct working that could lead to a correct solution with only one error 1 Mark for substantial correct working that could lead to a correct solutions
P4, P5, H7, H9	(d) (i) $8y = x^2 - 16$ $x^2 = 8y + 16$ $x^2 = 8(y + 2)$ $x^2 = (4)(2)(y + 2)$ This is a parabola of the form $(x - h)^2 = 4a(y - k)$, where the vertex is at (h, k) and the focal lenth is a. Therefore, the vertex is $(0, -2)$	2 Marks for correctly finding the vertex 1 Mark for substantial work that could lead to finding of the correct vertex

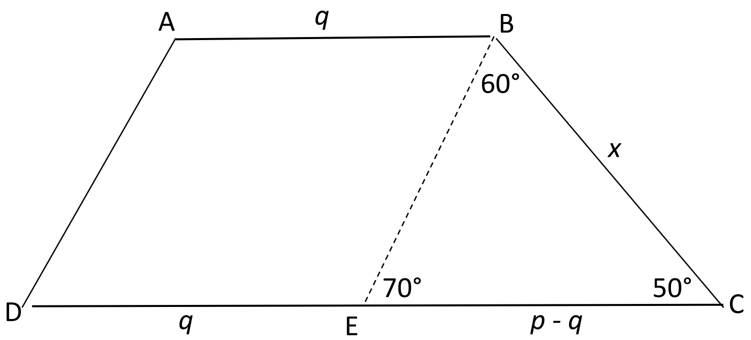
P4, P5, H7, H9	(ii) Focus is (0,0) Directrix is $y = -4$	2 Marks for correctly finding both the coordinates of the focus and the equation of the directrix 1 Mark for correctly finding one of them
P4, P5, H7, H9	(iii) 	2 Marks for complete correct graph showing focus, directrix, vertex and x-intercepts 1 Mark for drawing correct graph but missing a relevant feature.

Multiple Choice Answers	
1	D
2	D
3	A
4	C
5	B
6	D
7	B
8	C
9	B
10	A

Year 12 Trial	Mathematics Advanced	Examination 2017
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

Part	Solutions	Marking Guidelines
a.	$$2\cos^2\theta + 3\sin\theta\cos\theta + \sin^2\theta = 0$$ $$(2\cos\theta+\sin\theta)(\cos\theta+\sin\theta)=0$$ $2\cos\theta+\sin\theta=0$, $\sin\theta=-2\cos\theta$, $\frac{\sin\theta}{\cos\theta}=-2$, $\tan\theta=-2$, $\theta=116°34',\ 296°34'$ $\cos\theta+\sin\theta=0$, $\sin\theta=-\cos\theta$, $\frac{\sin\theta}{\cos\theta}=-1$, $\tan\theta=-1$, $\theta=135°,\ 315°$ $$\therefore\ \theta=116°34',\ 135°,\ 296°34',\ 315°$$	**Award 3 ~** Correct solution for *x* and *y* and correct reasoning **Award 2 ~** Correct solution for *x* and *y* **Award 1 ~** Makes some progress towards solution
b.	i) $l = r\theta$ Arc length of small circle $= l_{\text{small}} = \frac{5\pi}{6}\times 2 = \frac{5\pi}{3}$ cm Arc length of large circle $= l_{\text{large}} = \frac{\pi}{6}\times 3 = \frac{\pi}{2}$ cm Perimeter of the shaded region $= \frac{5\pi}{3} + \frac{\pi}{2} = \frac{13\pi}{6}$ cm	**Award 2 ~** Correct solution **Award 1 ~** Makes substantial progress towards solution
	ii) $A_{\text{shaded region}} = A_{\text{small segment}} + A_{\text{large segment}}$ $A = \left\{\frac{1}{2}\times 2^2\times\left(\frac{5\pi}{6}-\sin\frac{5\pi}{6}\right)\right\} + \left\{\frac{1}{2}\times 3^2\times\left(\frac{\pi}{6}-\sin\frac{\pi}{6}\right)\right\}$ $A = 2\left(\frac{5\pi}{6}-\sin\frac{5\pi}{6}\right) + \frac{9}{2}\left(\frac{\pi}{6}-\sin\frac{\pi}{6}\right)$ $A = 2\left(\frac{5\pi}{6}-\frac{1}{2}\right) + \frac{9}{2}\left(\frac{\pi}{6}-\frac{1}{2}\right)$ $A = \frac{29\pi-39}{12}\ \text{cm}^2$	**Award 3 ~** Correct solution **Award 2 ~** Makes substantial progress towards solution **Award 1 ~** Makes limited progress towards solution

c.	$\text{LHS} = \dfrac{1-\sin\theta}{\cos\theta}$ $= \dfrac{1-\sin\theta}{\cos\theta} \times \dfrac{1+\sin\theta}{1+\sin\theta}$ $= \dfrac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$ $= \dfrac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$ $= \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$ $= \dfrac{\cos\theta}{1+\sin\theta}$ $= \text{RHS}$	**Award 2 ~** Correct solution **Award 1 ~** Makes substantial progress towards solution
d.	(i) In ΔAOB, Let $\angle AOB = \theta$ $\cos\theta = \dfrac{24^2+35^2-40^2}{2(24)(35)}$ $\cos\theta = \dfrac{67}{560}$ $\theta = 83°$ $\therefore \angle AOB = 83°$ (ii) $A = \dfrac{1}{2}ab\sin C$ $= \dfrac{1}{2} \times 24 \times \sin 83°$ $= 417 \text{ km}^2$	**Award 2 ~** Correct solution **Award 1 ~** Makes substantial progress towards solution **Award 1 ~** Correct solution
e.	 Draw *BE* parallel to *AD*. $\dfrac{BC}{\sin 70°} = \dfrac{p-q}{\sin 60°}$ $\therefore BC = \dfrac{(p-q)\sin 70°}{\sin 60°}$	**Award 2 ~** Correct solution **Award 1 ~** Makes substantial progress towards solution

Outcomes Addressed in this Question

<table>
<tr><th>Part</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)</td><td>$y = 2xe^{3x}$
$\frac{dy}{dx} = 2e^{3x} + 2x\left(3e^{3x}\right)$
$= 2e^{3x} + 6xe^{3x}$</td><td>Award 2 marks for the correct answer.

Award 1 mark for substantial progress towards the solution</td></tr>
<tr><td>(b)</td><td>(i)
$Area\ of\ R_1 = \int_{-4}^{0} x^2(x+4)dx$
$= \int_{-4}^{0}\left(x^3 + 4x^2\right)dx$
$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_{-4}^{0}$
$= (0) - \left(64 - \frac{256}{3}\right)$
$= \frac{64}{3}\ units^2$</td><td>Award 2 marks for the correct answer.

Award 1 mark for substantial progress towards the solution</td></tr>
<tr><td></td><td>(ii)
$R_2 = \frac{64}{3}$
$\frac{64}{3} = \int_{0}^{2}\left(x^3 + 4x^2\right)dx + \frac{1}{2} \times 24 \times (b-2)$
$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_{0}^{2} + 12(b-2)$
$= \frac{2^4}{4} + \frac{4(2)^3}{3} + 12b - 24$
$= 12b - \frac{28}{3}$
$12b = \frac{64}{3} + \frac{28}{3}$
$12b = \frac{92}{3}$
$b = \frac{23}{9}$</td><td>Award 3 marks for the correct answer.

Award 2 mark for substantial progress towards the correct solution.

Award 1 mark for some progress towards the correct solution.</td></tr>
</table>

(c)	$\int_0^5 \frac{3}{2x+5}dx = \frac{3}{2}\times\int_0^5 \frac{2}{2x+5}dx$ $= \frac{3}{2}\left[\ln(2x+5)\right]_0^5$ $= \frac{3}{2}\left[\ln 15 - \ln 5\right]$ $= \frac{3}{2}\left[\ln 3\right] = \ln\left(3^{\frac{3}{2}}\right) = \ln\left(3\sqrt{3}\right)$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
(d)	$\frac{dy}{dx} = \frac{d}{dx}(\ln x)^2$ $= 2(\ln x)\times\frac{1}{x}$ $= \frac{2\ln x}{x}$ $\int \frac{2\ln x}{x}dx = (\ln x)^2 + c$ $\int_1^2 \frac{\ln x}{x}dx = \left[\frac{1}{2}(\ln x)^2\right]_1^2$ $= \frac{1}{2}\left[(\ln 2)^2 - (\ln 1)^2\right]$ $= \frac{1}{2}(\ln 2)^2$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
(e)	$y = \frac{1}{2}\left(e^x + e^{-x}\right)$ $y^2 = \frac{1}{4}\left(e^x + e^{-x}\right)^2$ $y^2 = \frac{1}{4}\left(e^{2x} + e^{-2x} + 2\right)$ $V = \pi\int_{-2}^{2}\frac{1}{4}\left(e^{2x} + e^{-2x} + 2\right)$ $V = \frac{\pi}{4}\left[\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x\right]_{-2}^{2}$ $V = \frac{\pi}{4}\left[\frac{e^4}{2} - \frac{e^{-4}}{2} + 4 - \frac{e^{-4}}{2} + \frac{e^4}{2} + 4\right]$ $V = \frac{\pi}{4}\left[e^4 - e^{-4} + 8\right]u^3$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

Question No. 15	Solutions and Marking Guidelines

Outcomes Addressed in this Question

P4 Chooses and applies appropriate arithmetic algebraic, graphical, trigonometric and geometric techniques.

Outcome	Solutions	Marking Guidelines
P4	a) (i) In triangles *ABM* and *APD* $\angle ABM = \angle APD = 90°$ (given, *ABCD* is a rectangle) $\angle BMA = \angle PAD$ (alternate angles in parallel lines *BC* & *AD*, given *ABCD* is a rectangle) $\therefore \Delta AMB$ is similar to ΔAPD (equiangular).	2 marks: correct solution 1 mark: substantial progress towards correct solution
	(ii) *BM* = 30. $\therefore$ *AM* = 50 (Pythagorean triad) As similar triangles, sides are in the same ratio. $\therefore \frac{PD}{40} = \frac{60}{50}$ $\therefore PD = 48$ cm	2 marks: correct solution 1 mark: substantial progress towards correct solution
P4	b) (i) *AC* has equation $y = -3x + 6$. $\therefore$ gradient *AC* = -3. gradient *BC* = $m(-18,0),(0,6) = \frac{6}{18} = \frac{1}{3}$. Since $mAC \times mBC = -3 \times \frac{1}{3} = -1$, $AC \perp BC$.	3 marks: correct solution 2 marks: substantially correct solution 1 mark: substantial progress towards correct solution
P4	(ii) Centre = midpoint $(-18,0),(4,-6) = (-7,-3)$.	
P4	Radius = $(4,-6),(-7,-3) = \sqrt{11^2 + 3^2} = \sqrt{130}$. Circle is $(x+7)^2 + (y+3)^2 = 130$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
	c) $y = \frac{1}{x}$ and $y = kx - 4$ meet when $kx - 4 = \frac{1}{x}$ $\therefore kx^2 - 4x - 1 = 0$. Since the line is a tangent to the curve, $\Delta = 0$. $\therefore (-4)^2 - 4k \times -1 = 0$ $\therefore 16 + 4k = 0$ $k = -4$.	1 mark: correct answer 1 mark: correct answer
P4	d) (i) The perpendicular distance of the point (x, y) to the line $3x - 4y + 1 = 0$ is 2 units. $\therefore \frac{\lvert 3x - 4y + 1 \rvert}{\sqrt{3^2 + (-4)^2}} = 2$ $\therefore \frac{\lvert 3x - 4y + 1 \rvert}{5} = 2$ $\therefore \lvert 3x - 4y + 1 \rvert = 10$ $\lvert 3x - 4y + 1 \rvert = 10$ is the two lines $3x - 4y + 1 = 10$ and $3x - 4y + 1 = -10$ i.e. the line $3x - 4y - 9 = 0$ and the line $3x - 4y + 11 = 0$.	2 marks: correct solution 1 mark: substantial progress towards correct solution
	(ii) The locus is two parallel lines.	1 mark: correct answer

Outcomes Addressed in this Question:
H5 applies appropriate techniques from the study of probability and series to solve problems

Outcome	Solutions	Marking Guidelines
H5	**(a) (i)** $T_n = 6n - 1$ **(ii)** $S_{100} = 30200$ **(b)** There are 8 multiples of 3: 24, 33, 36, 42, 45, 54, 63, 66 P(multiple of 3) = $\frac{8}{25}$ **(c)** $U_4 = ar^3 = \frac{15}{2}$ $\quad U_7 = ar^6 = 60$ $\therefore r^3 = 8$ i.e. $r = 2$ $\therefore 8a = \frac{15}{2}$ i.e. $U_1 = a = \frac{15}{16}$ **(d) (i)** $A_1 = P(1\cdot07) - 5000$ **(ii)** $A_2 = A_1(1\cdot07) - 5000$ $= P(1\cdot07)^2 - 5000(1\cdot07) - 5000$ $A_3 = P(1\cdot07)^3 - 5000(1\cdot07)^2 - 5000(1\cdot07) - 500$ $= P(1\cdot07)^3 - 5000(1\cdot07^2 + 1\cdot07 + 1)$ **(iii)** $A_8 = P(1\cdot07)^8 - 5000(1\cdot07^7 + 1\cdot07^6 + ... + 1\cdot07 + 1)$ $0 = P(1\cdot07)^8 - 5000\left(\frac{1(1\cdot07^8 - 1)}{1\cdot07 - 1}\right)$ $\therefore P(1\cdot07)^8 = 5000\left(\frac{1\cdot07^8 - 1}{0\cdot07}\right)$ $P = \$29856\cdot50$ **(e) (i)** $P(win) = \frac{0+1+2+3+4+5}{6\times6} = \frac{15}{36} = \frac{5}{12}$ **(ii)** $P(\text{Exactly one from two}) = P(WL) + P(LW)$ $= \frac{5}{12}\times\frac{7}{12} + \frac{7}{12}\times\frac{5}{12}$ $= \frac{70}{144}$ or $\frac{35}{72}$ **(iii)** $P(\text{At least 1 win from 5}) = 1 - P(\text{No wins})$ $= 1 - \left(\frac{7}{12}\right)^2 = \frac{95}{144}$	**(a)(i) 1 mark :** Correct answer **0 marks:** No simplification of formula on reference sheet. **(ii) 1 mark:** Correct answer **(b) 2 marks:** Correct answer. **1 mark:** One component of the fraction correct. **(c) 2 marks:** Correct answer **1 mark:** Correct ratio, or correct 'a' from incorrect ratio. **(d) (i) 1 mark:**: Correct answer. **(ii) 2 marks**: Correct answer with working. **1 mark**: Relevant progress. **(iii) 2 marks:** Correct value, to nearest dollar, with working. **1 mark:** Progress illustrating sum of series. **(e) (i) 1 mark:** Show **how** the outcomes sum to 15 **(ii) 2 marks:** Correct answer. **1 mark:** Significant progress. **(iii) 1 mark:** Correct answer.